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Canted spiral: An exact ground state of *XXZ* zigzag spin ladders

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We derive the exact ground states for a one-dimensional family of $S=1/2$ *XXZ* Hamiltonians on the zigzag ladder. These states exhibit true long-range spiral order that spontaneously breaks the $U(1)$ invariance of the Hamiltonian. Besides breaking a continuous symmetry in $d=1$, this spiral ordering has a ferromagnetic component along the symmetry axis that can take any value between zero and full saturation. In this sense, our canted spiral solutions are a generalization of the SU(2) Heisenberg ferromagnet to nonzero ordering wave vectors of the transverse spin components. We extend this result to the $d=2$ anisotropic triangular lattice.

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I. INTRODUCTION

The search for chiral phases in one-dimensional frustrated magnets has been very active during the last 10 years.[1–7] This interest was triggered by the prediction of a ground state with nonzero vector spin chirality, $\langle \mathbf{S}_j \times \mathbf{S}_{j+1} \rangle \neq 0$, for the J_1, J_2 *XXZ* chain with $|J_1| \ll J_2$ (J_1 and J_2 are the nearest- and next-nearest exchange interactions). The J_1, J_2 chain is equivalent to a zigzag ladder [see Fig. 1(a)] and it has also attracted a lot of interest during the last decades.[8,9] One of the main reasons for the continuing fascination generated by this model is in the interplay between geometric frustration and strong quantum fluctuations that leads to a rich and exotic variety of physical phenomena.

The vector chirality has to be distinguished from the scalar chirality, $\langle \mathbf{S}_{j-1} \cdot \mathbf{S}_j \times \mathbf{S}_{j+1} \rangle$, which breaks different discrete symmetries and is associated with different physical quantities.[10] As it is mentioned in Ref. 5, classical states with spontaneously broken chirality only exist together with helical long-range order. While the vector chirality distinguishes left and right spirals, the sign of the scalar chirality distinguishes between positive and negative canting angles for a given spiral orientation. The helical order breaks the continuous symmetry of global spin rotations along the z axis. Consequently, the existence of long-range helical order is in most cases precluded by zero point fluctuations of $d=1$ quantum systems.[11,12] On the other hand, chiral orderings are more robust against quantum fluctuations because they only break discrete symmetries. It is for this reason that the chiral orders of quantum systems can be thought as remnants of the helical order in classical systems. This observation has motivated the search for chiral orders in quantum spin Hamiltonians whose ground state exhibits helical order in the $S \to \infty$ limit.

The zigzag *XXZ* ladder is one of the simplest spin models whose classical counterpart has a helical ground state in a certain region of exchange parameters. The original proposal of a chiral ground state for the quantum version of this model was based on a mean field treatment of the bosonized Hamiltonian.[2] A similar approach was used by Kolezhuk and Vekua[5] to obtain a field induced chiral state in the Heisenberg (*XXX*) zigzag ladder. These mean field approaches were later validated by numerical simulations,[6,7] but there are some regions of the quantum phase diagram were the situation is still unclear.[7] In general, the numerical detection of these phases is very challenging due to the lack of commensuration between the dominant wave vector of the spin-spin correlator and the finite size chains that can be solved numerically.[13] It is therefore desirable to find exact analytical solutions for the chiral ground states and compute the most relevant correlation functions without making any approximation.

Here we consider a family of $S=1/2$ *XXZ* zigzag ladders for which we derive an exact ground-state subspace with a very unusual property: *it is the semi-classical version of a canted spiral phase* [see Fig. 1(b)]. This implies that the ground state has true long-range helical order; i.e., it breaks spontaneously the continuous $U(1)$ symmetry of global spin rotations along the z axis. Moreover, the ground state is still a canted spiral on finite size ladders for a discrete set of ratios, J_1/J_2, between the two exchange constants. The most curious of aspect of these solutions is that in contrast to the usual example of the SU(2) Heisenberg ferromagnet [the SU(2) symmetry is spontaneously broken at $T=0$], the order parameter of the canted spiral *does not commute* with the Hamiltonian. We will see that this observation is related to the emergence of an SU(2) symmetry in the ground-state subspace.[14] The nonzero scalar and vector chiralites are an

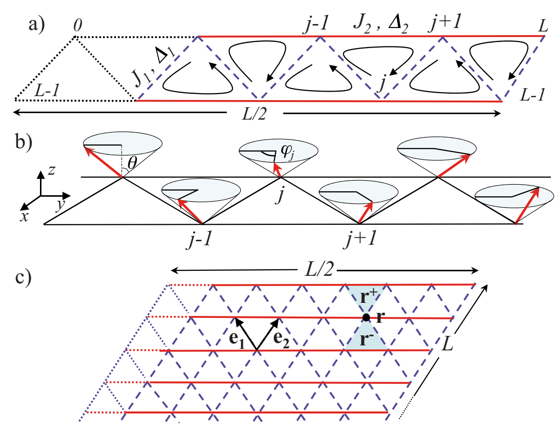

FIG. 1. (Color online) (a) Zigzag ladder. The dotted lines indicate that we are using periodic boundary conditions (PBCs). The arrows show the circulation of the spin and the electrical orbital currents that result from the nonzero vector and scalar chiralities of the canted spiral solution. (b) Canted spiral solution. The arrow at site j corresponds to $\langle \tilde{\Psi}_{\theta,\varphi_0} | \mathbf{S}_j | \tilde{\Psi}_{\theta,\varphi_0} \rangle$. (c) Extension of the zigzag ladder to $d=2$. $\mathbf{e}_1$ and $\mathbf{e}_2$ are primitive vectors. $\mathbf{r}_+$ and $\mathbf{r}_-$ denote the triangular plaquettes that are above and below the site $\mathbf{r}$.

epiphenomenon of the helical solution and they are computed in an exact way by exploiting the unentangled nature of the solution. We also provide an exact calculation of a branch of gapless single-magnon excitations with a quadratic low-energy dispersion around the spiral wave vectors $\pm Q$. Finally, we extend our exact canted spiral solution to the $d=2$ case of an *XXZ* Hamiltonian defined on an anisotropic triangular lattice [see Fig. 1(c)].

We start by considering an *XXZ* model on the zigzag ladder with L sites $(0 \leq j \leq L-1)$ depicted in Fig. 1,

$$H = \sum_{j,\nu} J^z_\nu \left(S^z_{j+\nu} S^z_j - \frac{1}{4} \right) + \frac{J_\nu}{2} (S^+_{j+\nu} S^-_j + S^-_{j+\nu} S^+_j), \tag{1}$$

where $\nu=1,2$, $0 \leq j \leq L-1$, $L \equiv 0$ (periodic boundary conditions), and $S^\pm_j = S^x_j \pm i S^y_j$. We will assume that the ratio of exchange constants satisfies the equation

$$\cos Q = -\frac{J_1}{4J_2} \tag{2}$$

where $Q = 2\pi n/L$ $(0 \leq n \leq L-1)$ is a wave vector in the Brillouin zone (BZ) of the ladder. We will also assume $J_2 > 0$, so Eq. (2) defines a set of $L/2+1$ different ratios J_1/J_2 for even L (the ratio is the same for $\pm Q$). In the thermodynamic limit, $L \to \infty$, Q can take any arbitrary value between 0 and π and Eq. (2) always has a solution for $|J_1| \leq 4J_2$.

We are particularly interested in the one-dimensional family of Hamiltonians defined by $J^z_\nu = \Delta_\nu J_\nu$ with

$$\Delta_1 = \cos Q = -\frac{J_1}{4J_2},$$

$$\Delta_2 = \cos 2Q = -1 + \frac{J_1^2}{8J_2^2}. \tag{3}$$

II. GROUND STATES

The next step is to provide a subspace of exact ground states for this Hamiltonian family. For this purpose we will define the vacuum, $|\varnothing\rangle = \otimes_j |\downarrow\rangle_j$, as the fully polarized state with all the spins down: $S^z_j|\varnothing\rangle = -(1/2)|\varnothing\rangle \quad \forall \quad 0 \leq j \leq L-1$. In addition, we introduce the spin operators in momentum space,

$$\mathcal{S}^z_q = \frac{1}{\sqrt{L}} \sum_j e^{iqj} S^z_j, \quad \mathcal{S}^+_q = \frac{1}{\sqrt{L}} \sum_j e^{iqj} S^+_j, \tag{4}$$

and $\mathcal{S}^-_q = (\mathcal{S}^+_q)^\dagger$. These operators obey the following commutation relations:

$$[\mathcal{S}^+_k, \mathcal{S}^-_q] = \frac{2}{\sqrt{L}} \mathcal{S}^z_{k-q}, \quad [\mathcal{S}^z_q, \mathcal{S}^+_k] = \frac{1}{\sqrt{L}} \mathcal{S}^+_{k+q}. \tag{5}$$

We claim that the set of linearly independent states

$$|\Psi_p\rangle = \sqrt{\frac{L^p (L-p)!}{p!\, L!}} (\mathcal{S}^+_Q)^p |\varnothing\rangle \tag{6}$$

are exact ground states of H for $0 \leq p \leq L$. The normalization prefactor, $\langle \Psi_p | \Psi_p \rangle = 1$, is obtained by noticing that $L^{p/2} (\mathcal{S}^+_Q)^p |\varnothing\rangle$ is a linear combination of $\binom{L}{p}$ states with coefficients that have the same absolute value equal to $p!$.

There are different ways of demonstrating our claim. Here we will use an algebraic procedure that unveils an emergent SU(2) symmetry of H. In first place, we will demonstrate that the states $|\Psi_p\rangle$ are degenerate eigenstates of H. For this purpose it is convenient to express H in momentum space,

$$H = \sum_q (J^z_q \mathcal{S}^z_q \mathcal{S}^z_{-q} + J_q \mathcal{S}^+_q \mathcal{S}^-_q), \tag{7}$$

where

$$J^z_q = J_1 \Delta_1 \cos q + J_2 \Delta_2 \cos 2q,$$

$$J_q = J_1 \cos q + J_2 \cos 2q. \tag{8}$$

From this expression we derive the following commutation relation by using Eqs. (3):

$$[S^+_Q, H] = \frac{i}{\sqrt{L}} \sum_{l=1,L} e^{ilQ} S^+_l a_l, \tag{9}$$

with

$$a_l = \sum_{\nu=1,2} J_\nu \sin \nu Q (S^z_{l+\nu} - S^z_{l-\nu}). \tag{10}$$

From Eq. (1), it is evident that

$$H|\varnothing\rangle = 0. \tag{11}$$

In addition, our definition of the vacuum state, $S^z_j|\varnothing\rangle = -(1/2)|\varnothing\rangle \quad \forall \quad 0 \leq j \leq L-1$, implies that $(S^z_j - S^z_l)|\varnothing\rangle = 0 \quad \forall \quad 0 \leq j, l \leq L-1$, and $a_l|\varnothing\rangle = 0$. Consequently,

$$[S^+_Q, H]|\varnothing\rangle = 0. \tag{12}$$

Finally, by using Eqs. (4) and (9) it is easy to verify that

$$[S^+_Q, [S^+_Q, H]] = 0. \tag{13}$$

The combination of Eqs. (12) and (13) leads to a very important result,

$$[S^+_Q, H](S^+_Q)^p|\varnothing\rangle = 0. \tag{14}$$

This result implies that S^+_Q is an infinitesimal generator of a continuous symmetry of H when H is restricted to the subspace $\mathcal{G}_Q$ generated by the states $|\Psi_p\rangle$ with $0 \leq p \leq L$,

$$[S^+_Q, P_{\mathcal{G}_Q} H P_{\mathcal{G}_Q}] = 0, \tag{15}$$

where $P_{\mathcal{G}_Q}$ is the projector on the subspace $\mathcal{G}_Q$. Moreover, Eqs. (11) and (14) imply that $H(S^+_Q)^p|\varnothing\rangle = 0$ or

$$H|\Psi_p\rangle = 0 \quad \forall \quad 0 \leq p \leq L. \tag{16}$$

This concludes our first step. The second step is to demonstrate that the eigenstates $|\Psi_p\rangle$ are *ground states* of H. Since the corresponding eigenvalues are equal to zero, we just need to demonstrate that H is semi-positive definite. This can be done by rewriting the Hamiltonian in the following way:

$$H = \sum_j H_j, \tag{17}$$

with $H_j = h_2(j-1, j+1) + h_1(j-1, j) + h_1(j, j+1)$

$$h_2(l,n) = J^z_2 \left(S^z_l S^z_n - \frac{1}{4} \right) + \frac{J_2}{2} (S^+_l S^-_n + S^-_l S^+_n)$$

$$h_1(l,n) = \frac{J_1^z}{2}\left(S_l^z S_n^z - \frac{1}{4}\right) + \frac{J_1}{4}(S_l^+ S_n^- + S_l^- S_n^+) \tag{18}$$

For Δ_1 and Δ_2 given by Eq. (3), we obtain that

$$H_j = (J_2 + J_1^2/8J_2)P_j \tag{19}$$

where P_j is a projector on the two-dimensional subspace $\mathcal{E}(j-1,j,j+1)$ generated by the states,

$$|\xi_{j\uparrow}\rangle = \gamma(2\cos Q S_{j-1}^+ S_{j+1}^+ - S_j^+ S_{j+1}^+ - S_{j-1}^+ S_j^+)|\varnothing\rangle_j,$$

$$|\xi_{j\downarrow}\rangle = \gamma(2\cos Q S_j^+ - S_{j-1}^+ - S_{j+1}^+)|\varnothing\rangle_j, \tag{20}$$

with $\gamma = 1/\sqrt{4+2\cos 2Q}$ and $|\varnothing\rangle_j = |\downarrow\rangle_{j-1}\otimes|\downarrow\rangle_j\otimes|\downarrow\rangle_{j+1}$. Given that $J_2 > 0$, the combination of Eqs. (17) and (19) implies that H is semi-positive definite. This concludes the demonstration of our main claim: *the states* $|\Psi_p\rangle$ generate a ground-state subspace $\mathcal{G}_Q$ of H. For $Q \neq 0,\pi$, there are two ground-state subspaces, $\mathcal{G}_Q$ and $\mathcal{G}_{\bar{Q}}$ ($\bar{Q} = -Q$), that correspond to right and left spiral solutions as we will see below. In addition to these solutions, there are other linearly independent ground states in the $\mathcal{S}^z = 0$ subspace that will be presented elsewhere.[15]

III. ORDER PARAMETER

The exact ground subspace $\mathcal{G}_Q$ contains a spiral order parameter that breaks spontaneously a continuous symmetry of H: this is the $U(1)$ symmetry of global spin rotations around the z axis. To see this we just need to choose an appropriate linear combination of the $|\Psi_p\rangle$ ground states,

$$|\tilde{\Psi}_{\theta,\varphi_0}\rangle = e^{-\theta\sqrt{L}/2(e^{i\varphi_0}\mathcal{S}_Q^+ - e^{-i\varphi_0}\mathcal{S}_Q^-)}|\Psi_L\rangle. \tag{21}$$

To verify that $|\tilde{\Psi}_{\theta,\varphi_0}\rangle \in \mathcal{G}_Q$ we just need to notice that $\mathcal{G}_Q$ is an invariant subspace of $\mathcal{S}_Q^+$ and $\mathcal{S}_Q^-$, something that will become evident when we discuss the emergent SU(2) symmetry of H. It is easy to verify that $|\tilde{\Psi}_{\theta,\varphi_0}\rangle = T_{Q,\varphi_0} R_\theta |\Psi_L\rangle$ with

$$R_\theta = e^{-\theta\sqrt{L}/2(\mathcal{S}_0^+ - \mathcal{S}_0^-)}, \quad T_{Q,\varphi_0} = e^{i\Sigma_j (jQ+\varphi_0)S_j^z}. \tag{22}$$

R_θ is the global spin rotation by an angle θ along the y axis, while T_{Q,φ_0} is the "twist" operator with momentum Q plus a global rotation by an angle φ_0 along the z axis. In other words, T_{Q,φ_0} rotates the spin j by an angle $\varphi_j = Qj + \varphi_0$ along the z axis. This implies that $|\tilde{\Psi}_{\theta,\varphi_0}\rangle$ is a canted spiral solution in which the spin j is fully polarized along the direction $\mathbf{u}_j = (-\sin\theta\cos\varphi_j, \sin\theta\sin\varphi_j, \cos\theta)$ [see Fig. 1(b)],

$$\mathbf{S}_j \cdot \mathbf{u}_j |\tilde{\Psi}_{\theta,\varphi_0}\rangle = \frac{1}{2}|\tilde{\Psi}_{\theta,\varphi_0}\rangle. \tag{23}$$

Consequently, $|\tilde{\Psi}_{\theta,\varphi_0}\rangle$ is a direct product state,

$$|\tilde{\Psi}_{\theta,\varphi_0}\rangle = \bigotimes_{j=0,L-1} |\tilde{\psi}_{\theta,\varphi_0}\rangle_j, \tag{24}$$

where

$$|\tilde{\psi}_{\theta,\varphi_0}\rangle_j = e^{i\varphi_j/2}\cos\theta/2|\uparrow\rangle_j + e^{-i\varphi_j/2}\sin\theta/2|\downarrow\rangle_j, \tag{25}$$

Besides breaking the continuous $U(1)$ symmetry of H, this solution breaks two discrete symmetries: spatial inversion, $\mathcal{I}$, and the product of time reversal times a π-rotation along the z-axis $\mathcal{T}e^{i\pi\sqrt{L}\mathcal{S}_0^z}$. The more physical implication is that the canted spiral carries nonzero vector, $\boldsymbol{\kappa}_j = \mathbf{S}_j \times \mathbf{S}_{j+1}\cdot\hat{\mathbf{z}}$, and scalar, $\boldsymbol{\chi}_j = \mathbf{S}_{j-1}\cdot\mathbf{S}_j\times\mathbf{S}_{j+1}$, spin chiralities,

$$\langle\tilde{\Psi}_{\theta,\varphi_0}|\boldsymbol{\kappa}_j|\tilde{\Psi}_{\theta,\varphi_0}\rangle = \frac{1}{4}\sin Q\sin^2\theta,$$

$$\langle\tilde{\Psi}_{\theta,\varphi_0}|\boldsymbol{\chi}_j|\tilde{\Psi}_{\theta,\varphi_0}\rangle = \cos\theta\sin^2\theta\sin^3(Q/2)\cos(Q/2). \tag{26}$$

These identities are easily obtained by exploiting the. direct product form of $|\tilde{\Psi}_{\theta,\varphi_0}\rangle$ [see Eq. (24)]: $\langle\mathbf{S}_j\times\mathbf{S}_l\rangle = \langle\mathbf{S}_j\rangle\times\langle\mathbf{S}_l\rangle$. The coexistence of both chiralities implies that there are spin and electric currents circulating around each of the triangular plaquettes [see Fig. 1(a)].[10,16]

IV. EMERGENT SU(2) SYMMETRY

A spontaneously broken continuous symmetry is uncommon for $d=1$ systems at zero temperature because it is usually prohibited by quantum fluctuations.[11] The simplest counter-example is the SU(2) ferromagnet ($Q=0$). In that case the continuous SU(2) symmetry is spontaneously broken because the order parameter, $\sqrt{L}(\mathcal{S}_0^x, \mathcal{S}_0^y, \mathcal{S}_0^z)$, coincides with the infinitesimal generators of the SU(2) symmetry group, i.e., it commutes with the Hamiltonian. Note that $\mathcal{S}_q^x = (\mathcal{S}_q^+ + \mathcal{S}_q^-)/2$ and $\mathcal{S}_q^y = i(\mathcal{S}_q^- - \mathcal{S}_q^+)/2$. The situation is less clear for the canted spiral under consideration ($Q\neq 0$) because the order parameter $\mathbf{P}_Q = (P_Q^x, P_Q^y, P_Q^z) = \sqrt{L}(\mathcal{S}_Q^x, \mathcal{S}_Q^y, \mathcal{S}_0^z)$ does not commute with H. However, we will show below that the components of the spiral order parameter generate an SU(2) group that is an *emergent* symmetry of H.[14]

The fact that the components of the canted spiral order parameter $\mathbf{P}_Q = (P_Q^x, P_Q^y, P_Q^z) = \sqrt{L}(\mathcal{S}_Q^x, \mathcal{S}_Q^y, \mathcal{S}_0^z)$ are elements of an SU(2) algebra results from the commutation relations (5),

$$[P_Q^\eta, P_Q^\mu] = i\epsilon_{\eta\mu\nu}P_Q^\nu, \tag{27}$$

where $\epsilon_{\eta\mu\nu}$ are the components of the Levi-Civita tensor. According to Eq. (15), $\mathbf{P}_Q$ commutes with $\mathcal{P}_{\mathcal{G}_Q} H \mathcal{P}_{\mathcal{G}_Q}$ and this implies that the SU(2) group generated by $\mathbf{P}_Q$ is an *emergent* symmetry of H.[14] Moreover, the ground-state subspace $\mathcal{G}_Q$ is an irreducible representation of this SU(2) group with an eigenvalue of the Casimir operator $\mathbf{P}\cdot\mathbf{P}|\Psi_p\rangle = L(L/4 + 1/2)|\Psi_p\rangle$ $\forall 0 \le p \le L$. This eigenvalue coincides with the square of the total spin for the uniform $Q=0$ case.

V. LOW-ENERGY EXCITATIONS

The $U(1)$ and the translational invariance of H imply that the single and two-magnon excitations on top of the fully polarized ground states, $|\varnothing\rangle$ and $|\Psi_L\rangle$, can also be computed in an exact way. In particular, the wave function for the single magnon with wave vector q on top of the fully polarized state $|\varnothing\rangle$ (the other one is obtained by applying the time reversal symmetry) is $\mathcal{S}_q^+|\varnothing\rangle$, and the corresponding energy eigenvalue is

$$\omega_q = \sum_{\nu=1,2} J_\nu(\cos\nu q - \cos\nu Q). \tag{28}$$

VI. TWO DIMENSIONS

Equation (19) shows that H is a sum of projectors P_j over all the triangular plaquettes of the zigzag ladder. This structure suggests a natural extension of H to the $d=2$ case of parallel chains coupled by zigzag bonds [Fig. 1(c)],

$$H^{2d} = \sum_{\mathbf{r}} H^{2d}_{\mathbf{r}^+} + H^{2d}_{\mathbf{r}^-} = (J_2 + J_1^2/8J_2)\sum_{\mathbf{r}} (P_{\mathbf{r}^+} + P_{\mathbf{r}^-}), \quad (29)$$

where

$$H^{2d}_{\mathbf{r}^\pm} = h_2(\mathbf{r} \pm \mathbf{e}_1, \mathbf{r} \pm \mathbf{e}_2) + h_1(\mathbf{r} \pm \mathbf{e}_1, \mathbf{r}) + h_1(\mathbf{r}, \mathbf{r} \pm \mathbf{e}_2) \quad (30)$$

and $P_{\mathbf{r}^\pm}$ is the projector on the subspace $\mathcal{E}(\mathbf{r}\pm\mathbf{e}_1, \mathbf{r}, \mathbf{r}\pm\mathbf{e}_2)$ [see Eq. (20)]. The sites $\mathbf{r}$ belong to the $L\times L/2$ lattice depicted in Fig. 1(c). Note that H^{2d} is an XXZ Hamiltonian with exchange interactions J_1^z, J_1 along the diagonal bonds and $2J_2^z$, $2J_2$ along the horizontal bonds (now each horizontal bond is common to two triangular plaquettes). Again, the boundary conditions are periodic. The three sites $(\mathbf{r}\pm\mathbf{e}_1, \mathbf{r}, \mathbf{r}\pm\mathbf{e}_2)$ are the corners of the triangles that are above (+) and below (−) the site $\mathbf{r}$ [see Fig. 1(c)].

The $d=2$ version of the canted spiral state is

$$|\tilde{\Psi}^{2d}_{\theta,\varphi_0}\rangle = \otimes_{\mathbf{r}} |\tilde{\psi}_{\theta,\varphi_0}\rangle_{\mathbf{r}}, \quad (31)$$

where $|\tilde{\psi}_{\theta,\varphi_0}\rangle_{\mathbf{r}}$ is given by Eq. (25) with φ_j replaced by $\varphi_{\mathbf{r}} = \varphi_0 + \mathbf{Q}\cdot\mathbf{r}$. $\mathbf{Q}$ is a wave vector of the BZ that satisfies

$$\mathbf{Q}\cdot\mathbf{e}_2 = -\mathbf{Q}\cdot\mathbf{e}_1 = Q. \quad (32)$$

Again Eq. (2) holds for $Q=2\pi n/L$. To prove that $|\tilde{\Psi}^{2d}_{\theta,\varphi_0}\rangle$ is a ground state of H^{2d} we simply use the right hand side of Eq. (29) and note that $|\tilde{\psi}_{\theta,\varphi_0}\rangle_{\mathbf{r}\pm\mathbf{e}_1} \otimes |\tilde{\psi}_{\theta,\varphi_0}\rangle_{\mathbf{r}} \otimes |\tilde{\psi}_{\theta,\varphi_0}\rangle_{\mathbf{r}\pm\mathbf{e}_2}$ belongs to $\mathcal{E}^\perp(\mathbf{r}\pm\mathbf{e}_1, \mathbf{r}, \mathbf{r}\pm\mathbf{e}_2)$. This argument also provides an alternative way of proving that $|\tilde{\Psi}_{\theta,\varphi_0}\rangle$ is a ground state in the $d=1$ case of H.

VII. CONCLUSIONS

In summary, we have demonstrated that the canted spiral solution is the ground state of a family of $S=1/2$ *XXZ* Hamiltonians defined on the zigzag ladder and parametrized by the wave vector Q of the spiral. The spontaneous breaking of a continuous symmetry is very unusual for $d=1$ quantum Hamiltonians when the order parameter does not commute with H. The case under consideration has an emergent SU(2) symmetry generated by the components of the canted spiral order parameter.

Our family of exact solutions includes the usual SU(2) ferromagnet in the $Q=0$ limit. Like in the ferromagnetic case, the $Q\neq 0$ solutions are product states (unentangled), meaning that inter-site quantum fluctuations that typically remove the long-range spiral order are absent in this case. The suppression of quantum fluctuations by geometric frustration is a rather unusual phenomenon. Moreover, the exact low-energy spectrum of single magnon excitations exhibits a quadratic dispersion around the ordering wave vectors $\pm Q$. In this sense, our canted spiral ground states are natural extensions of the ferromagnetic solution of isotropic Heisenberg models. Note also that our family of quasi-exactly solvable Hamiltonians includes cases which are arbitrarily close to the isotropic limit for $0<|Q|\ll\pi$ ($J_1 \lesssim -4J_2$); i.e., the exchange uniaxial anisotropy is a small perturbation. This regime corresponds to a ferromagnetic nearest-neighbor interaction $J_1<0$ (J_2 is always antiferromagnetic) that has also been considered in previous works.[17–20] It is important to mention that small perturbations will split the ground-state degeneracy and stabilize order parameters that only break discrete symmetries, such as chiral orderings or longitudinal ferromagnetism. In other words, we expect that chiral phases will appear in the proximity of the points with exact ground states that we have considered in this work.

Finally, we extended our exact solutions to the two-dimensional case of *XXZ* Hamiltonians defined on a spatially anisotropic triangular lattice [see Fig. 1(c)]. By showing that H can be written as a sum of projectors P_j over triangular plaquettes [see Eq. (19)], we are giving a prescription for constructing other higher-dimensional Hamiltonians with exact canted spiral ground states.

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